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Securitization, structuring and pricing of longevity risk

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ABSTRACT

Pricing and risk management for longevity risk have increasingly become major challenges for life insurers and pension funds around the world. Risk transfer to financial markets, with their major capacity for efficient risk pooling, is an area of significant development for a successful longevity product market. The structuring and pricing of longevity risk using modern securitization methods, common in financial markets, have yet to be successfully implemented for longevity risk management. There are many issues that remain unresolved for ensuring the successful development of a longevity risk market. This paper considers the securitization of longevity risk focusing on the structuring and pricing of a longevity bond using techniques developed for the financial markets, particularly for mortgages and credit risk. A model based on Australian mortality data and calibrated to insurance risk linked market data is used to assess the structure and market consistent pricing of a longevity bond. Age dependence in the securitized risks is shown to be a critical factor in structuring and pricing longevity linked securitizations.

1. Introduction

Longevity risk has become an increasingly important risk facing an increasing proportion of the world's population. Individuals would like to insure against this risk by purchasing life annuity products or other products with lifetime income guarantees. Annuity providers such as life insurance companies are unable to effectively manage aggregate longevity risk and are limited in capacity. Pension plans have increasingly offered defined contribution benefits with the risk of longevity remaining with individuals. Annuity providers' traditional methods for managing longevity risk have focused on participating policies, and financing through capital reserves. Reinsurers have been reluctant to accept the risk, some describing it as 'toxic' (Wadsworth, 2005). Financial markets have the potential to provide a risk pooling and risk management function for longevity risk. Securitization has been well developed for a range of risks including credit risk. Longevity bonds and related derivative contracts allow the securitization of the risk inherent in annuity portfolios leading to a more vigorous retail market in longevity risk management products.

People are living longer, yet more are retiring at younger ages. Labour participation rates for OECD males aged 60–64 have fallen from 70%–90% in the 1970s to 20%–50% today (Creighton et al., 2005). Individual future lifetimes are also becoming more variable (Booth et al., 2002; Morgan, 2007). These will result in an increased reliance on income sources including life annuities and lifetime income guarantee products to fund longer retirement time periods. Demand for individual annuity products will also be influenced by the shift from defined benefit (DB) to defined contribution (DC) pension plans. Defined contribution (DC) plans do not currently provide longevity protection (Creighton et al., 2005; Lin and Cox, 2005).

To illustrate the importance of developing a longevity market, the Australian pension (superannuation) industry had AUD 1177 billion in funds under management at December 31, 2007, two thirds of which were in defined contribution or hybrid DC/DB funds (APRA, 2007a). In contrast, the Australian lifetime annuity market was only AUD 3.9 billion in assets (APRA, 2007b). Purcal (2006) investigates the demand and supply constraints that have contributed to the size of this annuity market and concludes that on the supply side, longevity risk and the lack of long term debt instruments have prevented insurers from actively pursuing annuity business. On the demand side there has also been a shift towards investment linked products because of their flexibility and potential for higher returns. Similar issues arise in other developed economies.

Securitization has become an important technique for transferring illiquid risks into financial markets allowing risk pooling and risk transfer for many illiquid retail products such as house mortgages, corporate loans and life insurance policies. Mortgage and other asset-backed securities have been the main focus of securitization but increasingly the insurance market has been developed

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initially with the sale of rights to emerging profits from life insurance business (Cowley and Cummins, 2005). The transfer of credit risk via collateralized debt obligations (CDOs) has been a more recent development. The securitization of pure insurance risks began in the mid-1990s through insurance linked securitization (ILS) and the catastrophe bond market. This has since grown to over USD 5.6 billion worth of bonds issued in 2006 (Lane and Beckwith, 2007). Insurance risk securitization allows the transfer of pure insurance risk to investors. Securitization also provides an efficient alternative to insurance risk-transfer methods such as reinsurance.

The initial mortality risk securitization was the Swiss Re Vita Capital issue in December 2003. This mortality bond was designed to reduce the exposure of Swiss Re to catastrophic mortality deterioration over its three-year term (Blake et al., 2006a). Lane and Beckwith (2005, 2006) outline a number of other recent mortality linked issues by Swiss Re (Queensgate in 2005, and ALPS II in 2006) and Scottish Re (Orkney Holdings in 2005). These transactions securitize entire blocks of in-force business. Payments are indemnity based, linked to the actual experience of the cedent. These structures bundle underwriting, business, interest rate and mortality risks. Investors do not gain pure mortality exposures, a major contributor to the success of the Vita issues (Blake et al., 2006a).

The securitization of longevity risk was proposed using a 'survivor bond' (Blake and Burrows, 2001; Cox et al., 2000; Dowd, 2003; Blake, 2003). These bonds offer coupon payments linked to the survival rates of a reference population. Annuity providers can receive a payment stream that matches their liability profile, hedging longevity exposures. In November 2004 the European Investment Bank, advised by BNP Paribas, proposed the first survivor bond issue. Coupon payments were linked to a mortality index for English and Welsh males 65 years old and discounted at LIBOR minus 35 basis points, including a premium for the transfer of longevity risk. The exposure was underwritten by Partner Re through a series of longevity swaps. In late 2005 the bond was withdrawn for redesign. Blake et al. (2006a) provide a thorough analysis of the major concerns including an insufficient term to maturity of 25 years, excessive basis risk, model and parameter risk, and the capital intensive structure.

Securitization, structuring and pricing of longevity risk for a multi-age annuity portfolio is considered in this paper, extending (Lin and Cox, 2005; Liao et al., 2007). The impact of age dependence with a multiple-age portfolio is analysed. A tranche structure similar to that used in the collateralized debt obligation (CDO) market is assessed. Pricing models for longevity bonds include application of the Wang (1996, 2000, 2002) transform to a mortality distribution and the Lee and Carter (1992) model. This approach has been subject to criticism (see Cairns et al., 2006; Bauer and Russ, 2006) and an approach that falls within the framework of financial risk models is more appropriate. Dahl (2004) has developed financial risk models for mortality risk modelling. Risk adjusted probability measures can then be calibrated to market data. Although there is no currently active market in longevity risk, there are a number of existing mortality linked securities and a significant insurance linked security market that can provide price information for related risks.

The paper is structured as follows. Section 2 provides a background to longevity risk products and their pricing. In Section 3, a tranched longevity bond structure designed for an annuity portfolio with multiple ages is developed. The data and methodology used to price the longevity bond structure is covered in Section 4. Section 5 discusses the pricing and structuring and Section 6 concludes.

2. Longevity bonds background

Investment banks structure and issue securitized products. Longevity bonds are designed to transfer longevity exposure to the capital markets. Lin and Cox (2005), and others including Blake et al. (2006a,b,c) and Blake et al. (2006c), analyse longevity bonds. Coupon payments are contingent on a single-age reference index describing the number of annuitants initially aged x alive at time t, l_{x+t}. If this index, known as the loss measure, is greater than anticipated then the coupon paid to investors is reduced. The difference is paid to the bond issuer as compensation for higher annuity liabilities. These structures usually assume a single cohort and the impact of dependence between ages is not assessed. Only the coupons are at risk and the bond is designed to be issued in a single tranche.

Lane and Beckwith (2007) note that tranched issues are becoming increasingly popular in the insurance linked security market. In 2006 and 2007, new ILS issues were dominated by multi-tranche offerings. The tranche structures are based on CDOs and provide more tailored risk structures for capital markets investors. In the mortality bond market, the transactions since the Vita I issue have involved multiple tranches.

Chang and Shyu (2007) analyse the pricing of a tranched life insurance linked security under mortality dependence. Their 'Collateralized Insurance Obligation' is a product designed to transfer the mortality risk of a life insurance portfolio. It has the same underlying concept as the Swiss Re Vita Capital bonds. Mortality dependence between lives is incorporated using a Clayton copula, an approach that is popular in credit risk securitization. Mortality rates for age i are given by a linear mortality rate function:

$$\mu_i(t) = \mu_i^0(t) + \mu_i^g(t) + \mu_i^a(t) + \mu_i^l(t) \log\left[\frac{I(t)}{B(t)}\right], \tag{1}$$

where the first term is the base mortality rate, and the others allow for gender, age and income respectively. The price of each tranche is determined under a risk neutral measure. Their analysis suggests that Lin and Cox's (2005) independence assumption overestimates the premium of the equity tranche, and underestimates the premiums of the mezzanine and senior tranches. This result is consistent with the JP Morgan analogy of the 'correlation cat' for CDOs (Blum and Overbeck, 2006).

Liao et al. (2007) further examine tranching in mortality linked securities with a product designed to transfer longevity risk. They assume that mortality follows a non-mean reverting stochastic process for a single age as proposed by Luciano and Vigna (2005). If B is the level annuity payment per period, $X(t)$ is the actual number of survivors at time t and $\bar{X}(t)$ the expected number, then the loss on an annuity portfolio at t is defined as

$$l(t) = \begin{cases} B(X(t) - \bar{X}(t)) & \text{if } X(t) \geq \bar{X}(t) \\ 0 & \text{if } X(t) \leq \bar{X}(t). \end{cases} \tag{2}$$

A difficulty in structuring longevity bonds is defining the percentage cumulative loss on the portfolio. In a CDO or mortality bond, this is naturally the percentage of the portfolio that has defaulted or died by a certain time, as defaults and deaths only occur once. In longevity securitization, the number alive can exceed expectations consistently over a number of years. Liao et al. (2007) overcome this by defining the percentage cumulative loss based on the face value of the bond issued. They determine optimal tranche weights to match hypothetical market demands for expected loss exposures.

In order to price a longevity risk linked security, the underlying mortality risk process needs to be a risk adjusted pricing measure. In an incomplete market, such as the longevity risk market, equilibrium pricing theory can be used to derive a risk adjustment (Hull, 2003). Wang (1996, 2000, 2002) developed a framework for pricing risks that aimed to unify actuarial and financial theory based on the distortion operator

$$g_\lambda(u) = \Phi[\Phi^{-1}(u) - \lambda], \tag{3}$$

where the parameter λ is the 'market price of risk'. The distortion can be applied to a cumulative density function $F(t)$, to yield a 'risk adjusted' function $F^*(t) = g_\lambda(F(t))$. Both Lin and Cox (2005) and

Liao et al. (2007) employ this approach to determine a risk adjusted mortality measure. Market annuity data are used to calibrate λ. The major criticism of this approach is that it does not readily reflect different prices of risk across ages (Cairns et al., 2006; Bauer and Russ, 2006).

A number of ways of calibrating the risk adjusted probability measure have been proposed. Biffis (2005) suggests using basic insurance contracts traded in the secondary reinsurance market to imply a risk adjusted probability measure, but concedes that a deep market in such contracts does not exist. Blake et al. (2006c) incorporate the market price of risk as a parameter within their model and calibrate it to the BNP/EIB issue. Ballotta and Haberman (2006) assume that the market is risk neutral with respect to mortality risk mainly because of the inability to adequately assess the market risk premium. Lin and Cox (2005) use annuity data to calculate implied market probabilities for their model, an approach mirrored by Bauer and Russ (2006). As opposed to adjusting the underlying probability distribution, Lane (2000) has constructed an empirical model for pricing insurance linked securities where the spread on the security is a function of the expected losses of the issue (*EL*) and its expected excess return (*EER*). The *EER* is the 'risk loading' demanded by the market for accepting the exposure. To accommodate asymmetric loss distributions, Lane (2000) proposes using the probability of first loss (*PFL*) and the conditional expected loss (*CEL*) given exceedance of a tranche attachment point. Equivalent concepts in the credit and insurance risk literature are the probability of default (frequency) and the loss given default (severity). ILS tranches are priced using the functional form

$$EER = \gamma (PFL)^{\alpha} \times (CEL)^{\beta}. \tag{4}$$

Lane fits the model to insurance linked security (ILS) transactions to estimate γ, α and β. The model provides a reasonable fit for the market based *EER*. Lane and Beckwith (2005) use the model to evaluate the Swiss Re Vita Capital issue, and Lane and Beckwith (2006) review prices in the wind catastrophe bond market following the 2005 US hurricane season.

3. Structuring a longevity bond

The proposed structure for a longevity bond is based on that used for a collateralized debt obligation. Over the term of the bond the issuer pays a regular premium to the tranche investors. The tranche is 'triggered' by higher than anticipated longevity improvements. In this event, the investor forfeits a fraction of their prescribed capital to the issuer, as compensation for the issuer's incurred losses on an annuity portfolio. The payments are based on a specified population mortality index. In the following period, the premium is paid on a notional tranche principal that has been reduced by the incurred loss. The proposed longevity bond is structured in a number of tranches. This allows the risk profile of the bonds to be tailored to investor demands.

The underlying annuity portfolio consists of lives of differing ages. The population mortality process includes common factors that result in dependence between lives. There is both systematic and non-systematic longevity risk. Systematic risk is the risk associated with changes in the underlying (population) mortality rates. Non-systematic risk arises from the random variation in deaths in a portfolio for a fixed mortality rate. Systematic risk is not diversifiable, and thus does not decrease with the increasing size of a portfolio, whereas non-systematic risk is diversifiable. Longevity bonds aim to provide an alternative means of managing the systematic risk.

Payments are aggregated for all the lives in the portfolio and then allocated to tranches reflecting the seniority of the tranche. Fig. 1 summarizes the cashflows of the longevity bond where the issuer is assumed to have the obligation to pay fixed annuity payments of A to the survivors $l(x, t)$ of those initially aged x. The junior tranche is also referred to as the equity tranche.

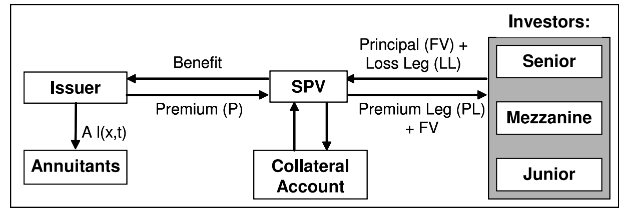

Fig. 1. The proposed tranched longevity bond.

Tranche investors subscribe an initial principal amount *FV* (in total), that is held as collateral in a special purpose vehicle (SPV). If no losses are incurred on the tranche, *FV* is returned to the investors at maturity. At the beginning of each period, investors receive a premium payment, *P*, from the issuer via the SPV. In exchange, a fraction of *FV* is transferred from the SPV to the issuer, in the event that longevity exceeds expectations. This provides compensation for higher than anticipated losses on the issuer's annuity portfolio. *FV* is not evenly reduced for all investors, since this depends on the allocation of portfolio loss between the tranches. After a loss has occurred, premiums are then calculated as a percentage of the reduced notional principal.

The price of each tranche is determined by equating the expected present values of premiums and losses under an equivalent risk adjusted probability measure for the mortality process.

3.1. Bond cash flows and annuity portfolio losses

The payments on the longevity bond are contingent on the losses on the underlying annuity portfolio. The longevity bond has a term to maturity of T periods and a total face value of FV. The bond's cashflows are determined by a reference annuity portfolio of $n(0)$ lives of different ages at time $t = 0$. It is assumed that annuitant i is paid a whole of life annuity of A_i per period, $i = 1, \dots, n(0)$. The indicator $I_i(t) = 1_{\tau_i > t}$ jumps from 1 to 0 at the time of death τ_i of an annuitant. The loss on the portfolio at time t is defined as

$$L(t) = \sum_{n(x,0)} (I_i(t)A_i - E\left[I_i(t)A_i\right])^+ \tag{5}$$

which is the amount by which the annuity payments at time t exceed the expected payments. Losses on the portfolio are not symmetric, giving rise to a number of 'option-like' features.

For an initial age of life i at time 0 of x_i, the probability of the life living to age $(x_i + t)$ is

$$ {}_tp_{x_i} = E[I_i(t)] \tag{6}$$

$$ = \exp\left[-\int_0^t \mu(x_i, s)\mathrm{d}s\right]. \tag{7}$$

The portfolio of annuitants have different ages, so denote the number of lives initially alive aged x by $l(x, 0)$, with

$$\sum_{\text{all } x} l(x, 0) = n(0). \tag{8}$$

The number of lives alive at time t, initially aged x, is denoted by $l(x, t)$. For a given population survival probability ${}_tp_x$, the distribution of the number alive at time t is binomial:

$$l(x, t) = \sum_{\text{all } i \text{ for age } x} I_i(t) \sim \text{ Binomial } (l(x, 0), {}_tp_x) \,|_tp_x. \tag{9}$$

The total variability in the portfolio is the unconditional variance of the compound binomial distribution:

$$\text{Var}[l(x, t)] = E[\text{Var}[l(x, t)|_tp_x]] + \text{Var}[E[l(x, t)|_tp_x]]. \tag{10}$$

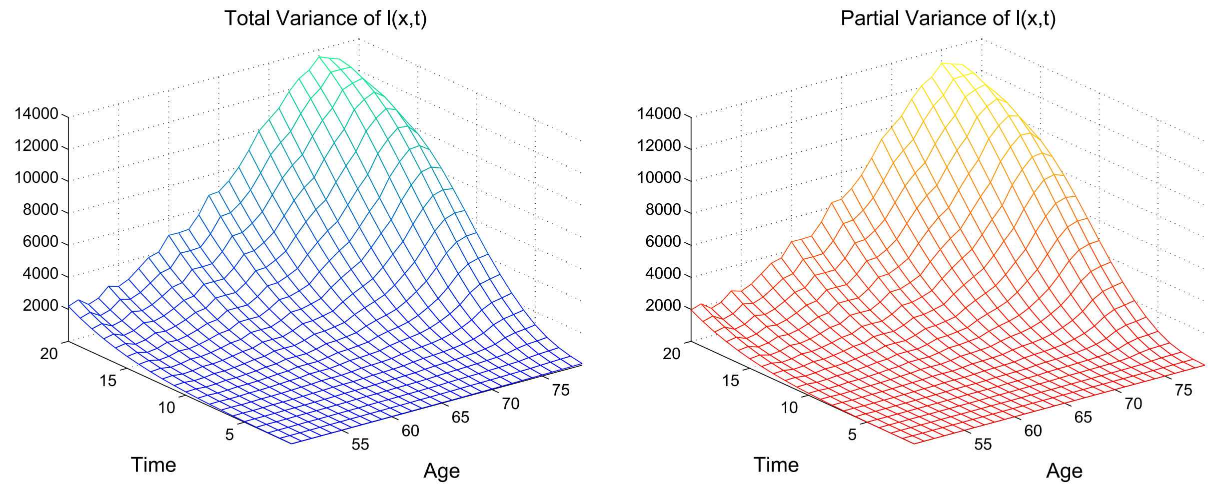

Fig. 2. Total and partial variance of $l(x, t)$.

For large portfolios of lives the main source of randomness will arise from systematic changes in the mortality rates impacting all lives in the portfolio. This randomness in $l(x, t)$ is due to the randomness in ${}_tp_x$. Fig. 2 shows both the total variance $\mathrm{Var}[l(x, t)]$ and the variance due to the random survival probability ${}_tp_x$, $\mathrm{Var}[E[l(x, t)|{}_tp_x]]$. The two plots are almost identical, so the volatility in $l(x, t)$ is primarily explained by the systematic variability of the underlying mortality process. Lin and Cox (2005) model a longevity bond based on longevity risk volatility from $l(x, t)$ given a fixed ${}_tp_x$, understating the longevity risk in a portfolio. Since the longevity bond aims to manage systematic longevity risk of the annuity portfolio, the systematic risk is the focus of this paper.

The portfolio loss in Eq. (5) can then be written as

$$L(t) = \left(A\sum_{\text{all x}} l(x,0)_t p_x - E\left[A\sum_{\text{all x}} l(x,0)_t p_x\right]\right)^+ \quad (11)$$

$$= \left(A\sum_{\text{all x}} l(x,0)_t p_x - A\sum_{\text{all x}} l(x,0)_t \bar{p}_x\right)^+ \quad (12)$$

where ${}_t\bar{p}_x$ is the expected value of the random survival probability ${}_tp_x$. The portfolio percentage cumulative loss can be written as

$$CL(t) = \frac{\sum_{s=1}^{t} L(s)}{FV}. \quad (13)$$

$CL(t)$ describes the percentage of the bond's face value that has been exhausted by portfolio losses up to that time. The choice of face value FV affects this loss, and in turn the risk profile of the bond. Unlike CDOs and Vita-style mortality bonds, it is possible to have $CL(t) > 1$, particularly for smaller values of FV. This arises when annuitants repeatedly exceed expectations of longevity whereas credit risky bonds and individual lives can default or die only once. Payments are restricted to cases where $CL(t) <= 1$.

3.2. Tranching by percentage cumulative loss

The J tranches are characterized by an attachment and detachment point denoted by $K_{A,j}$ and $K_{D,j}$ for $j = 1, \ldots, J$ respectively. The expected loss in each tranche determines the appropriate premium. The proposed tranche structure is based on the percentage cumulative loss of the portfolio with the longevity risk associated with the annuity portfolio tranched 'vertically', in a way that is similar to an 'excess of loss' reinsurance contract. As losses are incurred on the underlying annuity portfolio, they are allocated to a tranche when the cumulative loss falls between its attachment and detachment points. These points are expressed as a percentage of the bond's face value, FV, such that

$$\begin{aligned} &K_{A,1} = 0;\\ &K_{D,j-1} = K_{A,j};\\ &K_{A,j} < K_{D,j}; \quad \text{and,}\\ &K_{D,J} = 1. \end{aligned} \quad (14)$$

If cumulative losses exceed the detachment point of a tranche, it is retired and the losses are allocated to the next in order of seniority. The senior tranches will only attach if all subordinated ones have been retired. The cumulative loss on the jth tranche at time t is given by

$$CL_j(t) = \begin{cases} 0 & \text{if } L(t) < K_{A,j};\\ CL(t) - K_{A,j} & \text{if } K_{A,j} <= L(t) < K_{D,j};\\ K_{D,j} - K_{A,j} & \text{if } L(t) >= K_{D,j}, \end{cases} \quad (15)$$

where

$$CL(t) = \sum_{j=1}^{J} CL_j(t). \quad (16)$$

The expected percentage cumulative loss in tranche j at time t is

$$TCL_j(t) = \frac{E[CL_j(t)]}{K_{D,j} - K_{A,j}}. \quad (17)$$

Defining tranche payments by cumulative loss has a number of advantages. These can be seen by contrasting it with the approach used by Lin and Cox (2005), based on losses per period. In the latter case, the coupon paid to tranche investors only depends on the level of portfolio losses in each period. In a year of strong mortality improvement, coupons may be reduced to zero for a particular tranche. In the next year, if mortality rates return to expectation then coupons may be fully reinstated. The coupon payment stream thus becomes highly variable, reducing its attractiveness to investors. Initial capital costs would also be high, as the risk coverage per period is limited to the coupon size. Tranching by cumulative loss leads to a more predictable stream of cashflows. Once a tranche is exhausted, it will not be reinstated again. The tranches also provide greater coverage, as both principal and interest are at risk.

3.3. Pricing structured longevity risk

The price of a longevity bond tranche P_j is defined as a percentage of the principal at risk. This percentage is paid to the investor each period as the tranche premium. The fair price P_j^* is set so that the expected present value of the premium and claim

Table 1
Longevity bond structure.

Assumptions	
Bond face value:	FV = \$750,000,000.
Term to maturity:	$T = 20$ years.
Payment frequency:	Annually, for both premium and loss payments.
Number of tranches:	$J = 3$.
Initial age of annuitants:	$x = 50, \ldots, 79$.
Initial number of annuitants:	$n(0) = 60,000$. We assume that this is evenly distributed between the 30 ages, with $l(x, 0) = 2000\ \forall x$.
Annuity payments:	A = \$50,000 paid at the end of each year to each living annuitant.

payment legs of the tranche are equal. Each leg is a function of the expected percentage cumulative loss on the tranche at time t under a risk adjusted probability measure, denoted by $TCL_j^*(t)$. Assuming that premium payments occur at the beginning of each time period $t = 0, \ldots, T$, the value of tranche j's premium leg is equal to the risk adjusted expected present value of all premium payments to the investor:

$$PL_j = \sum_{t=1}^{T} P_j B(0, t-1)[1 - TCL_j^*(t-1)]. \tag{18}$$

The term $B(0, t-1)$ is the present value of a risk-free zero-coupon bond that pays \$1 at time $(t-1)$. $TCL_j^*(t-1)$ is the risk adjusted value of the expected percentage cumulative loss on the tranche at the time at which the premium is paid, so $[1 - TCL_j^*(t-1)]$ determines the proportional value of the notional face value of the tranche on which the premium is calculated. At the beginning of the contract, the premium is paid on 100% of the notional face value. This reduces over time to zero when the tranche is exhausted, and the premium payments cease.

The value of the claim payment leg is the risk adjusted expected present value of the loss payments, which occur at the end of each period:

$$LL_j = \sum_{t=1}^{T} B(0, t)[TCL_j^*(t) - TCL_j^*(t-1)]. \tag{19}$$

P_j^* is determined using a risk adjusted probability measure which incorporates a 'risk premium' for longevity risk.

The fair price of the tranche is then defined as the premium P_j^* such that

$$PL_j(P_j^*) - LL_j(P_j^*) = 0 \tag{20}$$

giving

$$P_j^* = \frac{\sum_{t=1}^{T} B(0, t)[TCL_j^*(t) - TCL_j^*(t-1)]}{\sum_{t=1}^{T} B(0, t-1)[1 - TCL_j^*(t-1)]}. \tag{21}$$

The pricing is based on a financial pricing mortality model in Wills and Sherris (2008) where the continuous time dynamics of the mortality rate are given under a risk adjusted probability measure $\mathbb{Q}$ by

$$\mathrm{d}\mu^{\mathbb{Q}}(x, t) = \left(a(x+t) + b + \sum_{i=1}^{N} \delta_{xi}\lambda_i(t)\right)\mu^{\mathbb{Q}}(x, t)\mathrm{d}t + \sigma\mu^{\mathbb{Q}}(x, t)\mathrm{d}W(x, t) \quad \text{for all } x \tag{22}$$

where $\underline{\lambda}(t) = [\lambda_1(t), \ldots, \lambda_N(t)]'$ is a vector of 'risk adjustments' and N is the number of factors driving volatility of the mortality rate. The expected changes in mortality rates differ by cohort and include a constant plus a cohort varying term. The 'prices of risk', $\lambda_i(t)$, can be calibrated to market price data where available and ILS data are used later to calibrate the model. The main motivation for using the model is the ability to capture the volatility of historical data through the random factors and the ability to include expected mortality changes varying by cohort.

Table 2
Attachment and detachment points as a percentage of bond face value (FV).

Tranche j	$K_{A,j}$ (%)	$K_{D,j}$ (%)
1	0	15
2	15	30
3	30	100

4. Implementation and analysis

Table 1 provides details of the structure for the longevity bond used for analysis. The FV determines the amount of coverage provided by the longevity bond issue. Losses are measured as a percentage of the bond's face value. The choices of FV, $n(0)$ and A were determined so that the longevity bond tranches have risk profiles commensurate with senior AAA, mezzanine BBB- and equity (unrated) securities. A long term to maturity of the bond is required to manage exposure to long term longevity improvements. The bond covers 30 ages in the portfolio over a 20 year term reflecting the longest dated Australian Government bonds. Other longer terms can be considered where a longer term bond market exists. In 2005, the French and UK governments issued 50 year bonds, which facilitate the management of interest rate risk over a longer time horizon. Longer term bonds will facilitate the market for longer longevity risk linked securities.

The attachment and detachment points of the jth tranche, $K_{A,j}$ and $K_{D,j}$, are defined in terms of the portfolio percentage cumulative loss and given in Table 2. The allocation of losses between tranches results in an unrated junior or equity tranche, a BBB-rated mezzanine tranche and a AAA rated senior tranche. The junior tranche should be retained by the issuer, managing moral hazard by aligning the issuer's interests with those of the investor. Where this does not occur, as happened in the CDO market, the tranche structure will not mitigate moral hazard. Tranching also creates a range of risk profiles, expanding the potential pool of risk capital to provide the funding.

4.1. Pricing tranched longevity risk

In an annuity portfolio the ages of the lives will vary and dependence between the ages is an important factor. In order to clearly demonstrate the impact of dependence, the longevity bond tranches are analysed under three specifications of dependence between ages in the underlying mortality process. These are independence, dependence based on principal components analysis (PCA) of Australian population mortality data and perfect dependence.

Data for calibrating and analysing the longevity bond were obtained from:

- Australian Population Mortality Data: ages 50–99, 1971–2004. Drawn from the Human Mortality Database, University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany).
- Australian Government Treasury Bill and Note Prices: maturity ranging from 12 months to 12 years. Drawn from the Bloomberg data service, 24/09/2007.